

Grant's Tutoring

LINEAR ALGEBRA and VECTOR GEOMETRY

Volume 2 of 2

September 2011 edition



This volume covers the topics taught
after your midterm exam.

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Know What You Need to Learn

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HOW TO USE THIS BOOK

I have broken the course up into lessons. Study each lesson until you can do all of my lecture problems from start to finish without any help. Then do the Practise Problems for that lesson. If you are able to solve all the Practise Problems I have given you, then you should have nothing to fear about your Midterm or Final Exam.

I have presented the course in what I consider to be the most logical order. Although my books are designed to follow the course syllabus, it is possible your prof will teach the course in a different order or omit a topic. It is also possible he/she will introduce a topic I do not cover. **Make sure you are attending your class regularly! Stay current with the material, and be aware of what topics are on your exam. Never forget, it is your prof that decides what will be on the exam, so pay attention.**

If you have any questions or difficulties while studying this book, or if you believe you have found a mistake, do not hesitate to contact me. My phone number and website are noted at the bottom of every page in this book. "Grant's Tutoring" is also in the phone book. **I welcome your input and questions.**

Wishing you much success,

Grant Skene

Owner of Grant's Tutoring

Lesson 9: Vectors

Important Vector Formulas and Facts:

- ✓ If \mathbf{v} is the vector (a, b, c) , which could also be denoted $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, then the length of \mathbf{v} (also called the norm of \mathbf{v}) is denoted $\|\mathbf{v}\|$, where:

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2 + c^2}$$

- ✓ Similarly, if $\mathbf{v} = (a, b, c, d)$, then $\|\mathbf{v}\| = \sqrt{a^2 + b^2 + c^2 + d^2}$. The pattern holds for vectors of any size, R^n .

- ✓ **A unit vector is a vector whose length is exactly 1 unit** (i.e., the norm of the vector is 1). Three standard unit vectors are \mathbf{i} , the unit vector in the positive x direction, \mathbf{j} , the unit vector in the positive y direction, and \mathbf{k} , the unit vector in the positive z direction.

- ✓ The unit vector in the direction of any vector \mathbf{v} can be denoted $\hat{\mathbf{v}}$, where:

$$\hat{\mathbf{v}} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$$

- ✓ Given two vectors, \mathbf{u} and \mathbf{v} , where the dot product $\mathbf{u} \cdot \mathbf{v} = 0$, then \mathbf{u} is **orthogonal** to \mathbf{v} (\mathbf{u} and \mathbf{v} make a right angle).

- ✓ The cosine of the angle, θ , between two vectors, \mathbf{u} and \mathbf{v} , is given by:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

- ✓ If $\mathbf{u} \cdot \mathbf{v}$ is **positive**, then $0^\circ < \theta < 90^\circ$. Which is to say, the angle, θ , between the two vectors, \mathbf{u} and \mathbf{v} , is an **acute angle**.

- ✓ If $\mathbf{u} \cdot \mathbf{v}$ is **negative**, then $90^\circ < \theta < 180^\circ$. Which is to say, the angle, θ , between the two vectors, \mathbf{u} and \mathbf{v} , is an **obtuse angle**.

- ✓ Given two vectors, \mathbf{u} and \mathbf{v} , the orthogonal projection of \mathbf{u} onto \mathbf{v} is denoted $\text{proj}_{\mathbf{v}} \mathbf{u}$, where:

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v}$$

- ✓ Given two vectors, \mathbf{u} and \mathbf{v} , the cross product, $\mathbf{u} \times \mathbf{v}$, produces a third vector, \mathbf{w} , which is orthogonal to both \mathbf{u} and \mathbf{v} . In fact, \mathbf{w} is normal to the entire plane containing \mathbf{u} and \mathbf{v} (i.e., \mathbf{w} makes a right angle with the \mathbf{u}, \mathbf{v} plane).

- ✓ $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ but $\mathbf{u} \times \mathbf{v} \neq \mathbf{v} \times \mathbf{u}$; however, $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$.

- ✓ The sine of the angle, θ , between two vectors, \mathbf{u} and \mathbf{v} , is given by:

$$\sin \theta = \frac{\|\mathbf{u} \times \mathbf{v}\|}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

- ✓ The area of a parallelogram with adjacent sides defined by the vectors \mathbf{u} and \mathbf{v} is given by:

$$\text{Area of a parallelogram} = \|\mathbf{u} \times \mathbf{v}\|$$

- ✓ The area of a triangle with two sides defined by the vectors \mathbf{u} and \mathbf{v} is given by:

$$\text{Area of a triangle} = \frac{1}{2} \|\mathbf{u} \times \mathbf{v}\|$$

- ✓ The volume of a parallelepiped with adjacent edges defined by the vectors \mathbf{u} , \mathbf{v} and \mathbf{w} is given by:

$$\text{Volume of a parallelepiped} = |(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$$

(Actually, you can pick any pair you want to compute the cross product with, then compute the dot product with the remaining vector.)

The important thing is do the cross product first, then the dot product.)

$$\text{Volume of the parallelepiped} = |(\mathbf{u} \times \mathbf{w}) \cdot \mathbf{v}| \text{ also, or}$$

$$\text{Volume of the parallelepiped} = |(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u}| \text{ also.}$$

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Let $\mathbf{u} = (3, -1, 4)$ and $\mathbf{v} = (-2, 1, -3)$. Find the following:
 - (a) $\|\mathbf{u}\|$
 - (b) $\mathbf{u} \cdot \mathbf{v}$
 - (c) $\mathbf{u} \times \mathbf{v}$
 - (d) $2\mathbf{u} - 3\mathbf{v}$
 - (e) The cosine of the angle between \mathbf{u} and \mathbf{v} .
 - (f) A unit vector in the direction of \mathbf{u} .
 - (g) The projection of \mathbf{u} onto \mathbf{v} .
 - (h) A vector, \mathbf{w} , which is orthogonal to \mathbf{u} and \mathbf{v} .
 - (i) A vector, \mathbf{m} , which is orthogonal to \mathbf{u} but not orthogonal to \mathbf{v} .
 - (j) The area of the triangle with adjacent sides \mathbf{u} and \mathbf{v} .

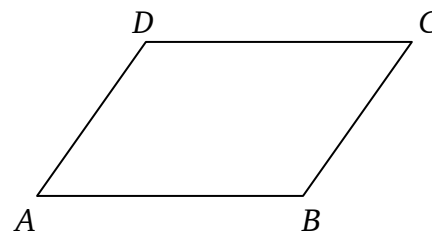
2. Let $\mathbf{v} = (2, 7, 1)$. Find a vector that is 10 units long, but pointing in the exact opposite direction to \mathbf{v} .

3.
 - (a) Let $\mathbf{v}_1 = (-1, 2, -2, -4)$, $\mathbf{v}_2 = (3, 1, 1, 3)$, and $\mathbf{v}_3 = (p, 3, q, 4)$. Find p and q such that \mathbf{v}_3 is orthogonal to both \mathbf{v}_1 and \mathbf{v}_2 .
 - (b) Let $\mathbf{u} = (1, 2, 3, 4)$ and $\mathbf{v} = (2, 5, 6, 7)$. Find a non-zero R^4 vector \mathbf{w} that is orthogonal to both \mathbf{u} and \mathbf{v} .

4. Given the three points $A = (2, 5, -2)$, $B = (1, 2, 3)$, and $C = (-6, 1, 4)$.
 - (a) Compute \overline{AB} and \overline{CB} .
 - (b) If $ABCD$ is a parallelogram, find the coordinates of point D .

5. Let the points $A = (0, 0)$, $B = (4, 0)$, $C = (6, 6)$, and $D = (2, 6)$ be the vertices of parallelogram $ABCD$. Use vectors to find the area of that parallelogram.

6. $ABCD$ is a parallelogram. Let $\mathbf{u} = \overline{AD}$ and $\mathbf{v} = \overline{AB}$.
 - (a) Write the vector \overline{AC} as a linear combination of the vectors \mathbf{u} and \mathbf{v} .
 - (b) Write the vector \overline{BD} as a linear combination of the vectors \mathbf{u} and \mathbf{v} .
 - (c) Use vector methods to show the diagonals of a parallelogram (lines AC and BD) intersect at right angles if and only if the parallelogram is a rhombus. (A rhombus is a parallelogram where all 4 sides are of equal length.)



Lesson 10: Lines and Planes

Equations of Lines:

To get the equation of a line, we need a **point on the line**, $\mathbf{p} = (x_0, y_0, z_0)$, and a **vector parallel to the line**, $\mathbf{v} = (a, b, c)$, then we can state the equation of the line.

☑ Parametric equations: $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$

☑ Vector form: $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ i.e. $(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Equations of Planes:

To get the equation of a plane, we need a **point on the plane**, $\mathbf{p} = (x_0, y_0, z_0)$, and a **vector normal to the plane**, $\mathbf{n} = (a, b, c)$, then we can state the equation of the plane.

☑ Standard form: $ax + by + cz = d$ (found by simplifying $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$)

☑ Point-normal form: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

The Distance between a Point and a Plane:

Given the plane $ax + by + cz = d$ and a point (x_1, y_1, z_1) not on the plane, to get the distance D between the point and the plane, first, rewrite the plane equation as $ax + by + cz - d = 0$, then you will substitute the point (x_1, y_1, z_1) into the left-hand side of this rewritten plane equation: $ax_1 + by_1 + cz_1 - d$.

$$\text{distance} = D = \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$$

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Find the vector form of the line passing through the point $(4, 1, 2)$ and parallel to the vector $3\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}$.
2. Find the parametric equations of the line passing through the point $(4, 1, 7)$ and parallel to the vector $5\mathbf{i} + 10\mathbf{j} + 12\mathbf{k}$.
3. Find the standard form of the plane containing the point $(1, 3, 5)$ and normal to the vector $4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$.
4. Let $A = (1, -2, 3)$, $B = (2, -3, 1)$ and $C = (3, -1, 2)$ be points in R^3 , find the following:
 - (a) The parametric equations of the line passing through A and B .
 - (b) The standard equation of the plane containing A , B and C .
5. Let $(x, y, z) = (2, 0, 1) + t(-3, 1, 2)$ be the equation of the line L in R^3 and let $x + 2y - z = -2$ be a plane Π in R^3 . Find the point of intersection of L and Π .
6. Find the equation of the plane which passes through the point $(-1, 5, 2)$ and is perpendicular to the line $(x, y, z) = (-1, 1, 0) + t(2, -1, 3)$.
7. Find the equation of the plane, in point-normal form, containing the point $(-1, -1, 2)$ and the line $x = 1 + 2t$, $y = -t$, $z = -2 + t$.
8. Find the point of intersection (if any) of the lines: $x = 1 + t$, $y = 2 - 3t$, $z = 4 + 2t$ and $x = 9 + 7s$, $y = 4 + 5s$, $z = 9 + 3s$.
9. Find the line of intersection of the two planes: $x + 2y - 3z = 5$ and $x + 3y + z = 9$.
10.
 - (a) Find the distance between the point $(2, 1, 4)$ and the plane $x + 2y - 3z = 5$.
 - (b) Verify that the planes $2x - 3y + 5z = 10$ and $4x - 6y + 10z = 13$ are parallel.
 - (c) Find the distance between the two planes in (b).

Lesson 11: Vector Spaces and Subspaces

The Definition of Subspace:

If W is a subset of vectors from a vector space V , then W is a subspace of V if and only if the following two conditions hold:

- (i) If \mathbf{v}_1 is a vector in W , then $r\mathbf{v}_1$ is also in W , for any real number r .
(W is closed under scalar multiplication.)
- (ii) If \mathbf{v}_1 and \mathbf{v}_2 are two vectors in W , then $\mathbf{v}_1 + \mathbf{v}_2$ is also in W .
(W is closed under addition.)

The 10 Axioms of Vector Spaces (I sincerely doubt you have to memorize these):

First, we define two mathematical operators: \oplus (which is the addition operator; this can be understood to be just like the good old “+” sign; it is telling you to add two things, but, we are now in a world where you might be given a totally new way to add; every time the \oplus is used, you must be told how you will “add” things in that world) and \odot (which is the multiplication operator; again this can be understood to be just like the good old “ \cdot ” sign telling you to multiply two things; but, once again, you have to be told exactly what way you are multiplying things in the given world). Simply put: \oplus is saying “plus” and \odot is saying “times”, but they may give you a wacky way to add or multiply things.

A vector space V is a set of vectors that satisfy the following 10 axioms. Let \mathbf{u} , \mathbf{v} and \mathbf{w} be three vectors in V and let a and b be any real numbers. (Your prof may write these in a different order and/or use different symbols.)

- (1) If \mathbf{u} and \mathbf{v} are two vectors in V , then $\mathbf{u} \oplus \mathbf{v}$ is also in V . (V is closed under the addition operator.)
- (2) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$.
- (3) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$.
- (4) For every vector \mathbf{v} there is a vector $\mathbf{0}$ in V such that $\mathbf{v} \oplus \mathbf{0} = \mathbf{0} \oplus \mathbf{v} = \mathbf{v}$.
- (5) For every vector \mathbf{v} there is a vector $-\mathbf{v}$ such that $\mathbf{v} \oplus -\mathbf{v} = \mathbf{0}$.
- (6) If \mathbf{v} is a vector in V , then $a \odot \mathbf{v}$ is also in V , for any real number a . (V is closed under the multiplication operator.)
- (7) $a \odot (\mathbf{u} \oplus \mathbf{v}) = a \odot \mathbf{u} \oplus a \odot \mathbf{v}$.
- (8) $(a + b) \odot \mathbf{v} = a \odot \mathbf{v} \oplus b \odot \mathbf{v}$.
- (9) $a \odot (b \odot \mathbf{v}) = (ab) \odot \mathbf{v}$.
- (10) $1 \odot \mathbf{v} = \mathbf{v}$.

(Note that (1) and (6) are the two conditions of subspace.)

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Prove whether the following subsets of the given vector space are subspaces.
 - (a) S is a subset of R^3 consisting of vectors in the form $(a, b, 2a + 3b + 1)$, where a and b are any real number.
 - (b) S is a subset of R^4 consisting of vectors in the form $(a, b, 2a + 3b, a - 2b)$, where a and b are any real number.
 - (c) S is a subset of M_{22} consisting of matrices in the form $\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$, where a, b and c are any real number.
 - (d) S is a subset of M_{22} consisting of matrices in the form $\begin{pmatrix} a & b \\ 0 & a^2 \end{pmatrix}$, where a and b are any real number.
 - (e) S is a subset of D_4 , all 4×4 diagonal matrices, where all the entries on the main diagonal are equal.
 - (f) S is a subset of P_4 consisting of all polynomials of the form $a + 2ax + bx^2 + 3bx^3 + x^4$, where a and b are any real number.
 - (g) S is a subset of P_2 consisting of all polynomials of the form $a + (a - 2b)x + bx^2$, where a and b are any real number.
 - (h) S is a subset of R^3 consisting of vectors with strictly integers in their components.

2. Determine whether the given subset of R^3 is a subspace of R^3 . Justify your answer.
 - (a) $\{(x, y, z) \mid y = 2z - x\}$
 - (b) $\{(x, y, z) \mid x^2 + y^2 = 9\}$

3. Determine whether the given subset of P_2 is a subspace of P_2 . Justify your answer.
 - (a) $\{p(x) \in P_2 \mid p(2) = 0\}$
 - (b) $\{p(x) \in P_2 \mid p(1) = 4\}$

4. Let F be a set of functions f such that $f(3) = 0$. Prove F is a subspace.
5. Let G be a set of functions g such that $g(0) = 3$. Prove G is not a subspace.
6. Let H be all 3×3 matrices A such that $\text{tr}(A) = 5$. Prove H is not a subspace.
7. Let W be all 3×3 matrices B such that $\text{tr}(B) = 0$. Prove W is a subspace.
8. Let Q be all 3×3 matrices C such that $C^T = -C$. Is Q a subspace? Justify your answer.
9. Let W be the set of all R^3 vectors. For any two vectors (x, y, z) and (x', y', z') in W and k in R , it is defined that:

$$(x, y, z) \oplus (x', y', z') = (x + x', y + y', 0)$$

$$k \odot (x, y, z) = (kx, ky, z)$$

- (a) Does W pass the commutativity of addition axiom for vector spaces? Which is to say, for all vectors \mathbf{u} and \mathbf{v} in W , does $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$?
- (b) Does W pass the existence of a zero vector axiom for vector spaces? Which is to say, for all vectors \mathbf{v} in W , is there a vector $\mathbf{0}$ in W such that $\mathbf{v} \oplus \mathbf{0} = \mathbf{0} \oplus \mathbf{v} = \mathbf{v}$?
- (c) Does W pass the distributive property axiom for vector spaces? Which is to say, for all vectors \mathbf{u} and \mathbf{v} in W , does $a \odot (\mathbf{u} \oplus \mathbf{v}) = a \odot \mathbf{u} \oplus a \odot \mathbf{v}$?
- (d) Does W pass the addition of scalar multiples property axiom for vector spaces? Which is to say, for all vectors \mathbf{v} in W , does $(a + b) \odot \mathbf{v} = a \odot \mathbf{v} \oplus b \odot \mathbf{v}$?
- (e) Does W pass the existence of the multiplicative identity axiom for vector spaces? Which is to say, does $1 \odot \mathbf{v} = \mathbf{v}$?

Lesson 13: Basis and Dimension

The Definition of Basis:

If V is any vector space (or subspace) and $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_n\}$ is a set of vectors in V , then S is called a basis of V if it satisfies the following two conditions:

- (i) S is linearly independent.
- (ii) S spans V .

✓ The dimension of any vector space V = the number of vectors in a basis for V .

Row Space, Column Space and Nullspace:

Given an $m \times n$ matrix A , we can consider the m rows to be m row vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_m$ and we can consider the n columns to be n column vectors $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \dots, \mathbf{c}_n$, then:

✓ The **row space** of A is the subspace of R^n spanned by $\{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_m\}$.

✓ The **column space** of A is the subspace of R^m spanned by $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \dots, \mathbf{c}_n\}$.

✓ The **nullspace** of A is the set of all solutions to $A\mathbf{x} = \mathbf{0}$.

✓ The dimension of the row space of A = the dimension of the column space of A
= the rank of A .

✓ The dimension of the nullspace of A is called the **nullity** of A .

✓ The rank of A + the nullity of A = n , the number of columns in A .

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Let $\mathbf{x} = (1, -1, 0, 1)$, $\mathbf{y} = (2, 1, 1, 0)$, and $\mathbf{z} = (-1, -5, -2, 3)$.

(a) Determine if \mathbf{x} , \mathbf{y} , \mathbf{z} are linearly dependent; if so, express one of them as a linear combination of the others.

(b) Find a basis for the subspace of R^4 spanned by $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$.

2. Prove that the set of vectors in the form $(a, b, a - 2b, 2a + 3b)$ is a subspace of R^4 and find a set of vectors that spans this subspace.

3. Find a basis and the dimension of the solution space of:

$$\begin{aligned} x_1 - x_2 + 2x_3 + 2x_4 &= 0 \\ x_1 + x_3 - x_4 &= 0 \end{aligned}$$

4. Given $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 & -4 \end{pmatrix}$, find a basis for the nullspace of A .

5. Do the vectors $(2, 1, 1)$, $(3, 0, -1)$ and $(4, 5, 2)$ form a basis for R^3 ?

6. Do the vectors $(2, 4, 2)$, $(1, 5, 1)$ and $(3, -7, 3)$ span R^3 ?

7. Do the vectors $1 - 2x + 5x^2$, $3 + 5x$ and $2 - 4x^2$ form a basis of P_2 ?

8. Do the vectors $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ form a basis of D_3 ?

9. Let S be the subspace spanned by the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ where $\mathbf{v}_1 = (1, -2, 0, 3)$, $\mathbf{v}_2 = (2, -5, -3, 6)$, $\mathbf{v}_3 = (0, 1, 3, 0)$, $\mathbf{v}_4 = (2, -1, 4, -7)$, $\mathbf{v}_5 = (5, -8, 1, 2)$.

- (a) Is the set linearly independent? Justify your answer.
- (b) Find a subset of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ that is a basis for S .
- (c) What is the dimension of S ?
- (d) Express the vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ that are not in your basis found in (b) as linear combinations of these basis vectors.

10. Given $A = \begin{bmatrix} 9 & -27 & 3 & 9 & 0 \\ -8 & 24 & -3 & -8 & 0 \\ 3 & -9 & 1 & 3 & 0 \\ 6 & -18 & -21 & 6 & 1 \end{bmatrix}$, then $B = \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is the reduced row

echelon form of A . Answer the following questions:

- (a) Find a basis for the column space of A .
- (b) Find a basis for the row space of A .
- (c) Find a basis for the nullspace of A .
- (d) What is the rank and nullity of A ?
- (e) Find a basis for the column space of B .
- (f) Find a basis for the row space of B .
- (g) Find a basis for the nullspace of B .