If you ever have a trig integral that features the product of two sine functions, product of two cosine functions, or the product of a sine and a cosine function where the **angles are different**, such as

$$\sin(5x)\cos(2x)$$
$$\sin(7x)\sin(2x)$$
$$\cos(8x)\cos(5x)$$

You must use a **Product Identity** to transform the product into a sum of two trig functions instead.

Memorize these identitites:

$$\sin A\cos B = \frac{1}{2}\sin(A+B)+\frac{1}{2}\sin(A-B)$$
$$\cos A\sin B = \frac{1}{2}\sin(A+B)-\frac{1}{2}\sin(A-B)$$
$$\sin A\sin B = \frac{1}{2}\cos(A-B)-\frac{1}{2}\cos(A+B)$$
$$\cos A\cos B = \frac{1}{2}\cos(A-B)+\frac{1}{2}\cos(A+B)$$

So, for example, I can transform $\sin(2x)\cos(5x)$ like so:

$$\sin(5x)\cos(2x) = \frac{1}{2}\sin(5x+2x)+\frac{1}{2}\sin(5x-2x) = \frac{1}{2}\sin(7x)+\frac{1}{2}\sin(3x)$$

I can transform $\sin(7x)\sin(2x)$ like so:

$$\sin(7x)\sin(2x) = \frac{1}{2}\cos(7x-2x)-\frac{1}{2}\cos(7x+2x) = \frac{1}{2}\cos(5x)-\frac{1}{2}\cos(9x)$$

I can transform $\cos(8x)\cos(5x)$ like so:

$$\cos(8x)\cos(5x) = \frac{1}{2}\cos(8x-5x)+\frac{1}{2}\cos(8x+5x) = \frac{1}{2}\cos(3x)+\frac{1}{2}\cos(13x)$$