

### **The Mean Value Theorem:**

Given the function  $f$  which is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ , then there exists at least one number  $c$  in  $(a, b)$  such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

In order to “verify” the Mean Value Theorem, for a given function  $f(x)$ , and given endpoints  $a$  and  $b$ , do the following:

**Step 1:** Sub  $a$  into the given function and  $b$  into the given function in order to compute  $f(a)$  and  $f(b)$ , respectively.

**Step 2:** Now you can compute the right-hand side of the Mean Value Theorem:  
$$\frac{f(b) - f(a)}{b - a}$$

**Step 3:** Compute the derivative of the function,  $f'(x)$ .

**Step 4:** Sub  $c$  in place of  $x$  in  $f'(x)$  (literally replace all the  $x$ 's with  $c$  in the derivative).

**Step 5:** Set  $f'(c)$ , your answer in Step 4, equal to  $\frac{f(b) - f(a)}{b - a}$ , your answer in Step 2.  
Solve for  $c$ .

**Step 6:** If you get more than one answer for  $c$ , only keep the solutions that lie between  $a$  and  $b$  (not including  $a$  and  $b$  themselves). The Mean Value Theorem predicts there will be at least one  $c$  in  $(a, b)$ , so you must get at one least answer for  $c$  that falls between  $a$  and  $b$ . If you do get at least one answer for  $c$ , you have verified the Mean Value Theorem.